1997: PART A

1) $V = 3t^2 - 2t - 1,\ t \geq 0$

a) $x(t) = \int V(t)\,dt = \int (3t^2 - 2t - 1)\,dt$

$= t^3 - t^2 - t + C$

$x(2) = 5 \quad \therefore \quad 5 = 2^3 - 2^2 - 2 + C$

$C = 3$

$x(t) = t^3 - t^2 - t + 3$

b) Average velocity $= \frac{1}{3}\int_0^3 V(t)\,dt = 5$

or: Av. velocity $= \frac{\text{total distance}}{\text{total time}}$

$= \frac{x(3) - x(0)}{3 - 0}$

$= \frac{18 - 3}{3} = 5$

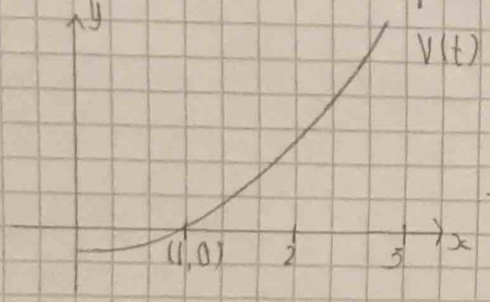
Instantaneous velocity $= 3t^2 - 2t - 1$

$3t^2 - 2t - 1 = 5$

$3t^2 - 2t - 6 = 0$

$t = 1.786$ to 3.d.p in $[0,3]$

c)

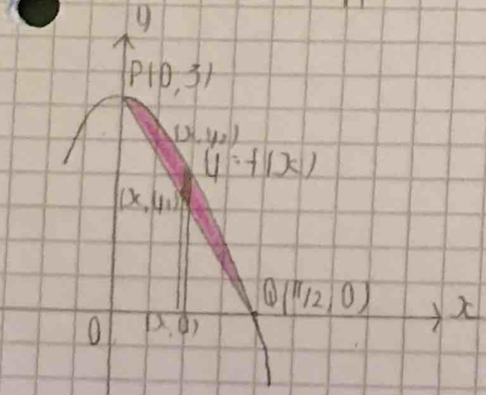


Total distance $= -\int_0^1 (3t^2 - 2t - 1)\,dt + \int_1^3 (3t^2 - 2t - 1)\,dt$

$= 1 + 16$

$= 17$

2)



$f(x) = 3\cos x$

a) Slope $PQ = \frac{0 - 3}{\pi/2 - 0} = -\frac{3}{\pi/2} = -\frac{6}{\pi}$

$y - 3 = -\frac{6}{\pi}(x - 0)$

$\pi y - 3\pi = -6x$

$6x + \pi y = 3\pi$

b) $Q\left(\frac{\pi}{2}, 0\right)$, $f'(x) = -3\sin x$

$f'\left(\frac{\pi}{2}\right) = -3\sin\frac{\pi}{2} = -3$

Equation of tangent:

$y - 0 = -3(x - \pi/2)$

$y = -3x + \frac{3\pi}{2}$

$3x + y = \frac{3\pi}{2}$

c) using mean value theorem:

$f'(c)\,Av = \frac{f(\pi/2) - f(0)}{\pi/2 - 0} = \frac{3\cos\pi/2 - 3\cos 0}{\pi/2} = -\frac{3}{\pi/2} = -\frac{6}{\pi}$ $\quad 0 < c < \frac{\pi}{2}$

$\therefore -3\sin c = -\frac{6}{\pi}$

$\sin c = \frac{2}{\pi}$

$c = \sin^{-1}\left(\frac{2}{\pi}\right) = 0.690$ to 3.d.p

d) $R(x) = y_2 - 0 = 3\cos x$

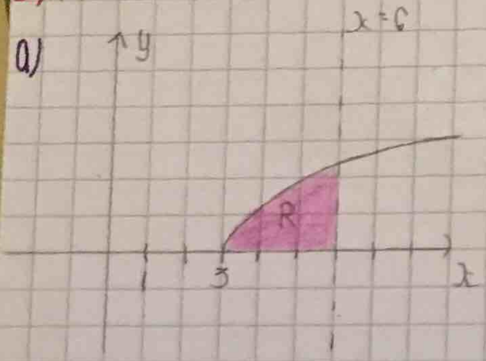
$r(x) = y_1 - 0 = \frac{3\pi - 6x}{\pi}$

$\text{volume} = \pi\int_0^{\pi/2} (3\cos x)^2 - \left(\frac{3\pi - 6x}{\pi}\right)^2 dx$

$= \pi\int_0^{\pi/2}\left[9\cos^2 x - \frac{(3\pi - 6x)^2}{\pi^2}\right]dx$

3) $f(x) = \sqrt{x - 3}$

a)



b) $\text{Area } R = \int_3^6 (x-3)^{1/2}\,dx$

$= \left[\frac{2}{3}(x-3)^{3/2}\right]_3^6$

$= \frac{2}{3}(3)^{3/2} - 0$

$= \frac{2}{3}\sqrt{27} = \frac{2}{3}(3\sqrt{3}) = 2\sqrt{3}$

$$= \int_{3}^{w} (x-3)^{1/3}\,dx$$

$$) A'(w) = \frac{d}{dw} \int_{6}^{w} (x-3)^{1/3}\,dx$$

$$A'(w) = (w-3)^{1/3}$$

$$A'(6) = (6-3)^{1/3}$$

$$= \sqrt{3}$$

1. PART B

4) $f(x) = x^3 - 6x^2 + p$

a) $f'(x) = 3x^2 - 12x$
$f'(x) = 0$
$3x^2 - 12x = 0$
$3x(x-4) = 0$
$x(x-4) = 0$
$x = 0, x = 4$

$f'(x)$: +   -   +
0   4
/   \   _   /

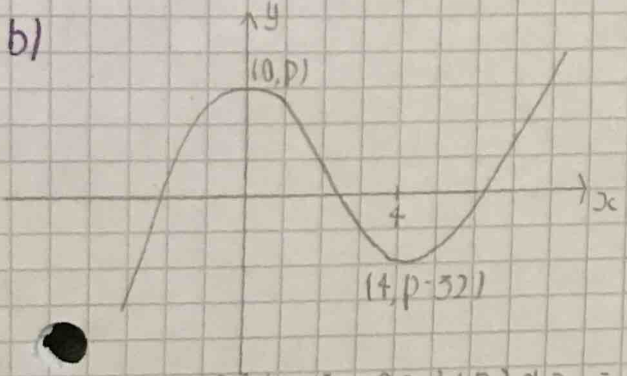
$f(0) = p$
$f(4) = 4^3 - 6(4^2) + p = -32 + p$

Rel. max at $x = 0$ since sign of $f'(x)$ changes from +ve to -ve and rel min at $x = 4$ since sign of $f'(x)$ changes from -ve to +ve

Min @ $(4, p-32)$, Max @ $(0, p)$

$\therefore f_{min} = p - 32$, $f_{max} = p$

b)



$f(x)$ has 3 distinct real roots if:

$p > 0 \quad \cap \quad p - 32 < 0$
$p < 32$

$0 < p < 32$

c) $\frac{1}{2-(-1)} \int_{-1}^{2} (x^3 - 6x^2 + p)\,dx = 1$

$\frac{1}{3}\left[\frac{x^4}{4} - 2x^3 + px\right]_{-1}^{2} = 1$

$\frac{1}{3}\left[4 - 16 + 2p - (1/4 + 2 - p)\right] = 1$

$\frac{1}{3}\left(-12 + 2p - \frac{1}{4} - 2 + p\right) = 1$

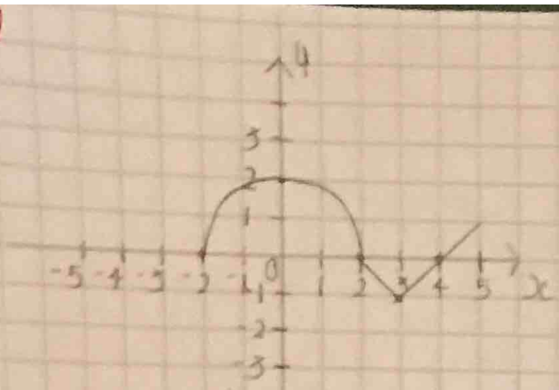
$\frac{1}{3}\left(-\frac{57}{4} + 3p\right) = 1$

$3p - \frac{57}{4} = 3$

$3p = \frac{69}{4}$

$p = \frac{69}{12} = \frac{23}{4}$

5)



$g(x) = \int_0^x f(t)\,dt$

a) $g(3) = \int_0^3 f(t)\,dt = -\frac{1}{2}(1)(1) + \frac{1}{4}\pi(2^2)$

$= -\frac{1}{2} + \frac{1}{4}(4)\pi$

$= \pi - \frac{1}{2}$

b) $f(x) = g'(x)$

$g'(x)$: -2 (+) 2 (−) 4 (+) 5

/ \ _ /

Rel. Max at $x = 2$ since $g'(x)$ changes sign from +ve to −ve at $x = 2$

c) $g'(3) = -1$, $\left(3, \pi - \frac{1}{2}\right)$

Equation of tangent:

$y - \left(\pi - \frac{1}{2}\right) = -1(x - 3)$

$y - \pi + \frac{1}{2} = -x + 3$

$y = -x + \frac{5}{2} + \pi$

$y + x - \pi - \frac{5}{2} = 0$

d) $g''(x)$: -2 (+) 0 (−) 2 (−) 3 (+) 5 $g''(x)$ undefined when $x = 2$ since $g'(x)$ doesn't exist at $x = 2$

Points of inflection @ $x = 0, x = 3$ since graph of $g'(x)$ changes from increasing to decreasing at $x = 0$ and decreasing to increasing at $x = 3$

6) $\frac{dv}{dt} = -2v - 32$, $v(0) = -50$

a) $\int \frac{dv}{-2v - 32} = dt$

$-\frac{1}{2}\ln|-2v - 32| = t + c$

$= -50$

$-\frac{1}{2}\ln|100-32| = c$

$\frac{1}{2}\ln(68) = c$

$\therefore -\frac{1}{2}\ln|-2v-32| = t - \frac{1}{2}\ln 68$

$-\frac{1}{2}\ln|-2v-32| + \frac{1}{2}\ln 68 = t$

$\frac{1}{2}\ln\left|\frac{68}{-2v-32}\right| = t$

$\ln\left|\frac{-68}{2v+32}\right| = 2t$

$\frac{-68}{2v+32} = e^{2t}$

$-68 = (2v+32)e^{2t}$

$2v+32 = -68e^{-2t}$

$v = \frac{-68e^{-2t} - 32}{2}$

$v = -34e^{-2t} - 16$

b) $\lim_{t\to\infty} v(t)$

$= \lim_{t\to\infty} (-34e^{-2t} - 16)$

$= \lim_{t\to\infty} \left(\frac{-34}{e^{2t}} - 16\right)$

$= 0 - 16$

$= -16$ ft/sec

c) $-16 - 34e^{-2t} = -20$

$34e^{-2t} = 4$

$e^{-2t} = \frac{2}{17}$

$-2t = \ln\left(\frac{2}{17}\right)$

$t = \frac{\ln(2/17)}{-2}$

$= 1.070$ seconds